Section I
Attempt Questions 1-10
All questions are equal value.
Use the multiple choice answer sheet for Questions 1-10

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1 Evaluated to three significant figures, $1-e^{-0.1}$ is…

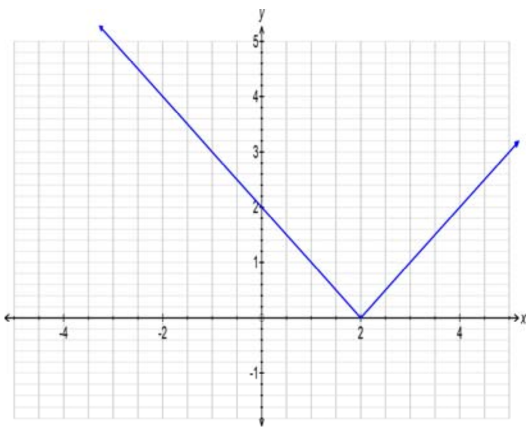
(A) 0.095 (B) 0.0952 (C) 0.0951 (D) -0.105

2 Which of these is a simplified version of $\dfrac{16-a^2}{8-2a}$?

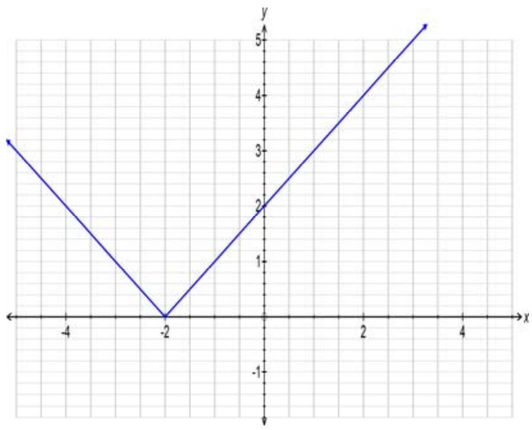
(A) $\dfrac{4-a}{2a}$ (B) $2-a$ (C) $\dfrac{4+a}{2}$ (D) $2-\dfrac{a}{2}$

3 Which of these best represents the graph of $y=|x-2|$?

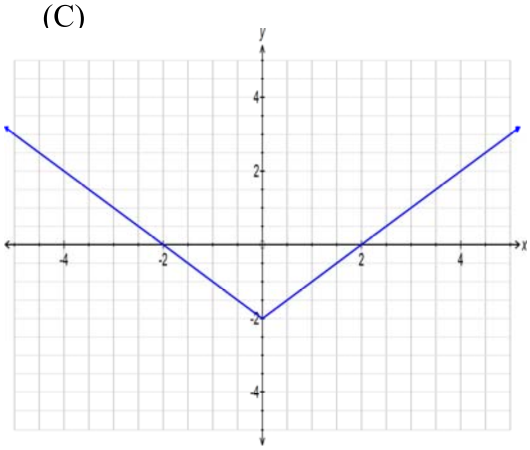
(A)



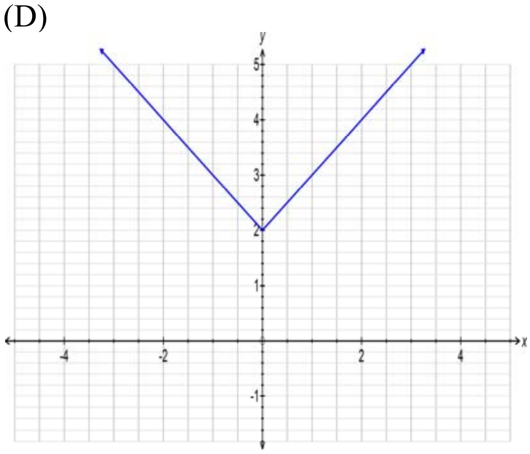
(B)



(C)



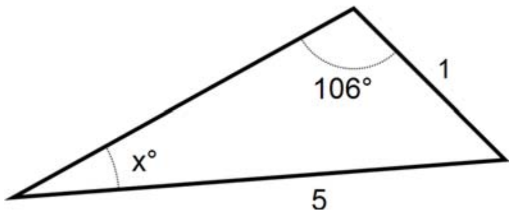
(D)



4 Which of these is the limiting sum of the geometric series $\frac{2}{5} - \frac{2}{15} + \frac{2}{45} - \frac{2}{135} + ...$

(A) $\frac{3}{5}$ (B) $\frac{8}{27}$ (C) 0 (D) $\frac{3}{10}$

5 Which of the following would you use to calculate the value of x in the diagram.



NOT TO SCALE

(A) $x = \sin^{-1}\left(\frac{\sin 106^\circ}{5}\right)$ (B) $x = \cos^{-1}\left(\frac{\sin 106^\circ}{5}\right)$

(C) $x = \sin^{-1}\left(5\sin 106^\circ\right)$ (D) $x = \sin^{-1}\left(\frac{1}{5}\right)$

6 Which of these represents the derivative of $x\sin x$ with respect to x.

(A) $\cos x$ (B) $-\cos x$ (C) $x\sin x + \cos x$ (D) $x\cos x + \sin x$

7 A bag contains yellow and blue marbles in the ratio of $5:2$. If a marble is selected at random, what is the probability it is blue?

(A) $\frac{1}{5}$ (B) $\frac{5}{7}$ (C) $\frac{2}{5}$ (D) $\frac{2}{7}$

8 If $\ln a = \ln b + \ln c$, then which of these is true?

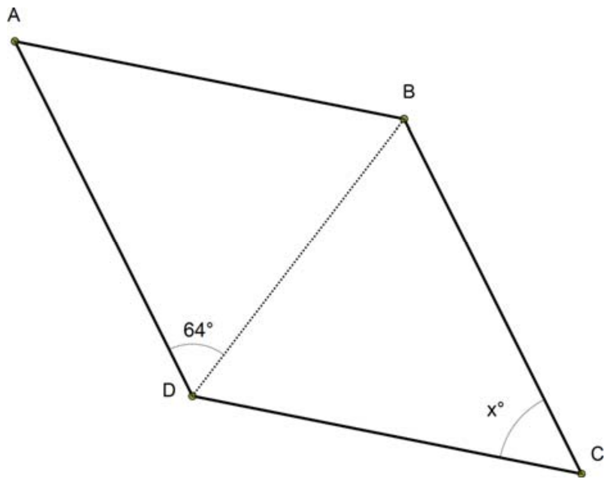
(A) $a = bc$ (B) $a = b + c$ (C) $\ln a = bc$ (D) $a = \frac{b}{c}$

9 Find the integers a and b such that $\left(3 - \sqrt{5}\right)^2 = a + b\sqrt{5}$.

(A) $a = 4 \quad b = 0$ (B) $a = 14 \quad b = -6$

(C) $a = 9 \quad b = -2$ (D) $a = 14 \quad b = 6$

10 The polygon ABCD below is a rhombus.



Which of these is the value of the angle x° marked on the diagram?

(A) 26 (B) 52 (C) 62 (D) 64

End of Section I

Section II
Attempt Questions 11-16
Each question is worth 15 marks.

Answer each question in a new writing booklet. Extra booklets are available.

All necessary working should be shown in every question

Question 11 (15 Marks) Use a NEW writing booklet.

(a) Consider a circle of radius $3cm$.

(i) Find the exact length of an arc that subtends an angle of 80° at the centre. **1**

(ii) Find, in radians, the angle at the centre which would create a sector of area $3cm^2$. **2**

(b) Find the derivative of $y = \ln(3x-1)$ with respect to x. **2**

(c) Find $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 x \, dx$ **2**

(d) A student had two weeks (14 days) to read a novel of 630 pages. He decided he would read 7 pages the first day, 10 pages the second day, 13 the third day and continue to increase the number of pages by 3 each day.

(i) How many pages did he read on the 14th day? **1**

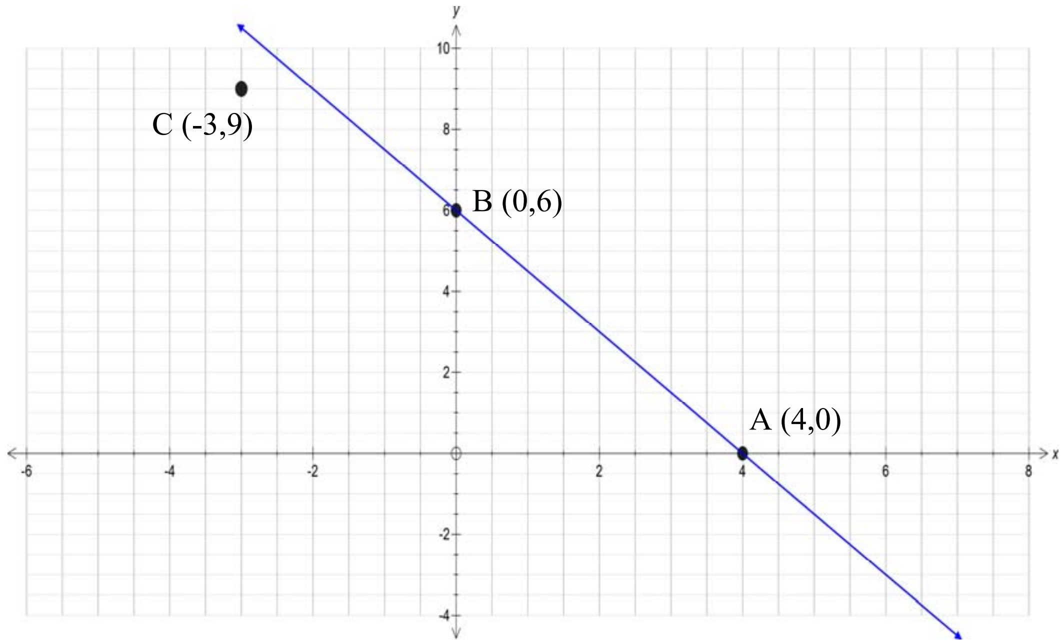
(ii) How many pages did he read in the course of the two weeks? **1**

(iii) If he decided to start by reading p pages on the first day and **increase** the number of pages read each day by p pages, find the value of p so he finished reading the book in 14 days. **3**

(e) Find the equation of the normal to the curve $y = e^{2x}$ at the point where $x = -2$. **3**

Question 12 (15 Marks) Use a NEW writing booklet.

(a) The diagram below shows a line through the points A (4,0) and B (0,6). It also shows the point C (–3,9).



(i) Find the length of the interval AB in simplest surd form. **2**

(ii) Find the gradient of the line through AB. **1**

(iii) Show that the equation of the line l through C parallel to AB has the equation $2y+3x-9=0$. **2**

(iv) Show that the point D (1,3), which forms the parallelogram ABCD, lies on the line l. **1**

(v) Find the perpendicular distance from D to AB. **2**

(vi) Find the area of the parallelogram ABCD. **1**

(b) The following table gives the values of a function $y = f(x)$ for three values of x.

x	3.1	3.3	3.5
y	4	5.6	3.2

Use these values and Simpson's Rule to estimate $\int_{3.1}^{3.5} f(x)\,dx$. **2**

(c) In a class there are 12 boys and 18 girls. A committee of three is chosen from the class.

(i) What is the probability that the committee comprises three girls? **1**

(ii) What is the probability that the committee contains at least one boy? **1**

(iii) What is the probability there are more boys than girls on the committee? **2**

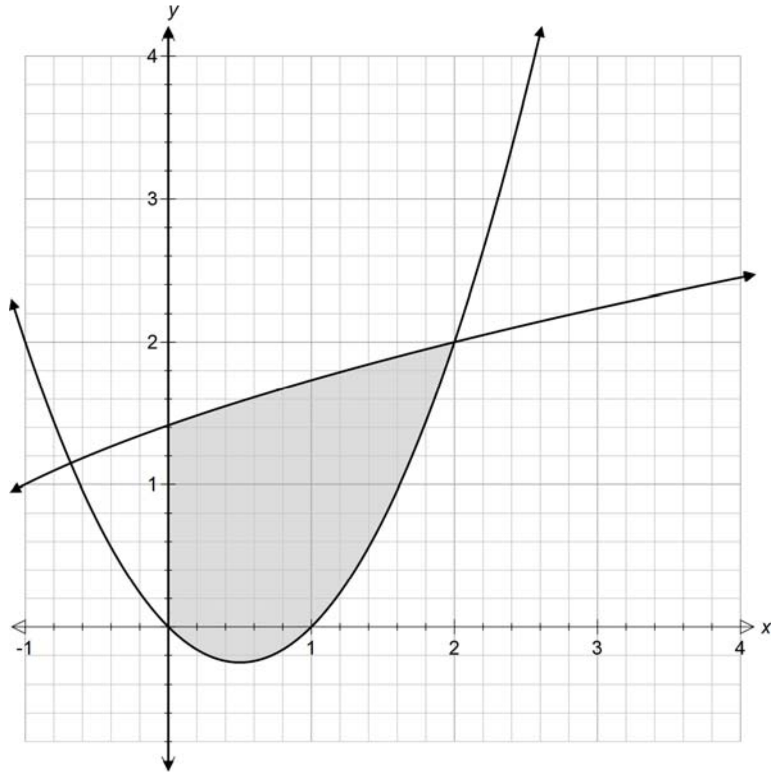
Question 13 (15 Marks) Use a NEW writing booklet.

(a) The roots of the quadratic $px^2 - x + q = 0$ are -2 and 5. **2**

Find the values of p and q.

(b) Evaluate $\sum_{n=0}^{4}(-2)^n$ **2**

(c) The diagram below shows the graphs of $y = \sqrt{x+2}$ and $y = x^2 - x$.



Find the area of the shaded region bounded by these two curves and the y-axis. **3**

(d) Consider the function $y = x^2 e^x$

(i) Show that $\frac{dy}{dx} = x(x+2)e^x$ **2**

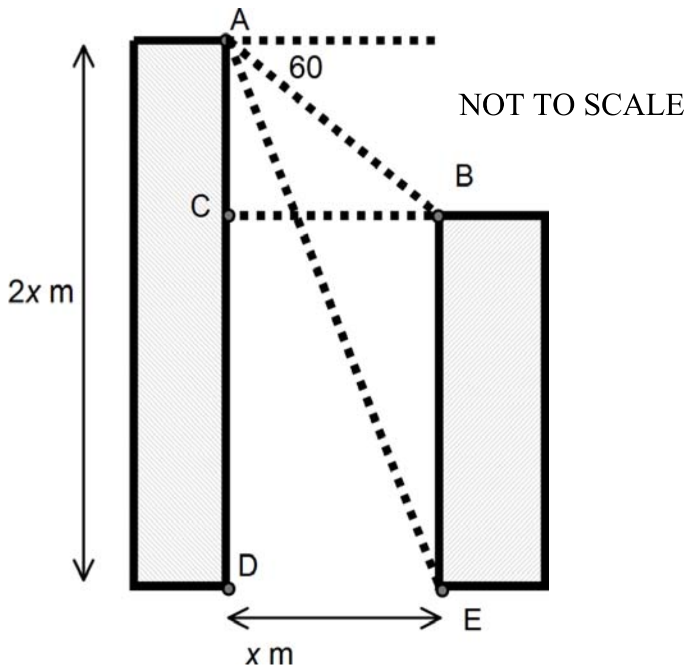
(ii) Find the turning points of this function and determine their nature. **3**

(iii) Find any intercepts with the coordinate axes. **1**

(iv) By considering the graph as $x \to \pm\infty$ or otherwise, sketch the function showing the turning points and intercept clearly. DO NOT CALCULATE POINTS OF INFLEXION **2**

Question 14 (15 Marks) Use a NEW writing booklet.

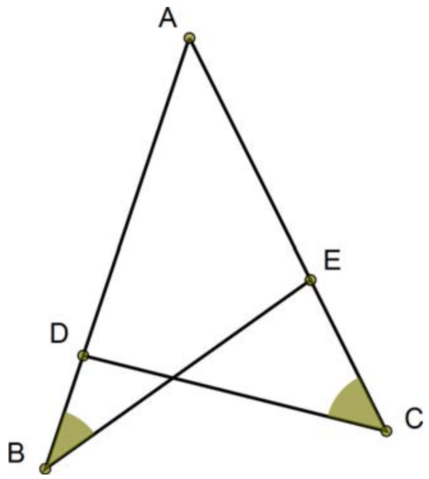
(a) The diagram below shows two office buildings. The taller building is $2x$ metres high and they are x metres apart. The angle of depression from the top of the taller tower to the top of the smaller is 60°.



(i) Find the distance AE from the top of the taller building to the base of the smaller in terms of x. **1**

(ii) Find the angle of depression of E from A to the nearest degree. **3**

(b) In the diagram D is a point on AB such that $AD = 4$ and $DB = 1$. E is a point on AC such that $AE = x$ and $EC = 8$. $\angle ABE = \angle ACD$.

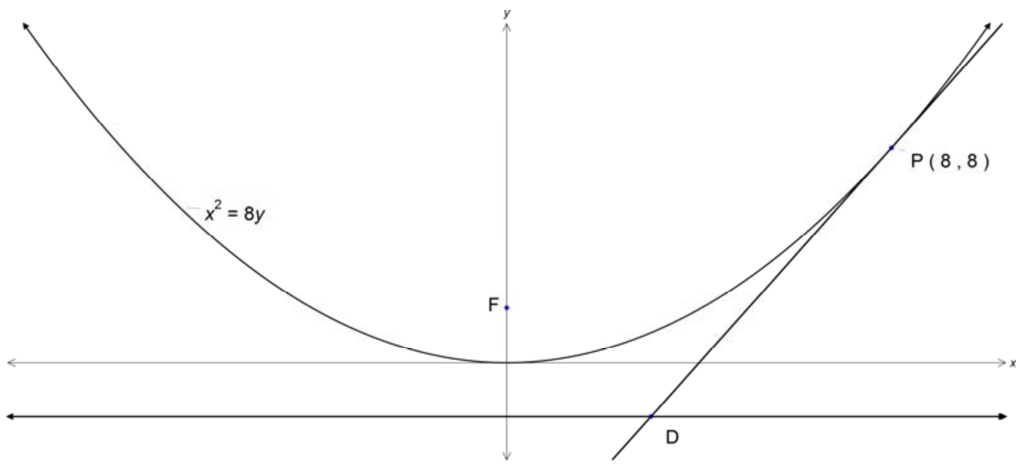


NOT TO SCALE

(i) Prove that ΔABE is similar to ΔACD **2**

(ii) Find x. **2**

(c) P (8,8) is a point on the parabola $x^2 = 8y$. F is the focus of the parabola. The tangent to the parabola at P meets the directrix at D.



(i) Show that the equation of the tangent at P has equation $2x - y - 8 = 0$ **2**

(ii) Show that the point D has the coordinates (3,-2) **1**

(iii) Show that $\angle PFD = 90^\circ$ **2**

(d) Find $\int \frac{e^{2x}}{e^{2x}+1}\,dx$ **2**

Question 15 (15 Marks) Use a NEW writing booklet.

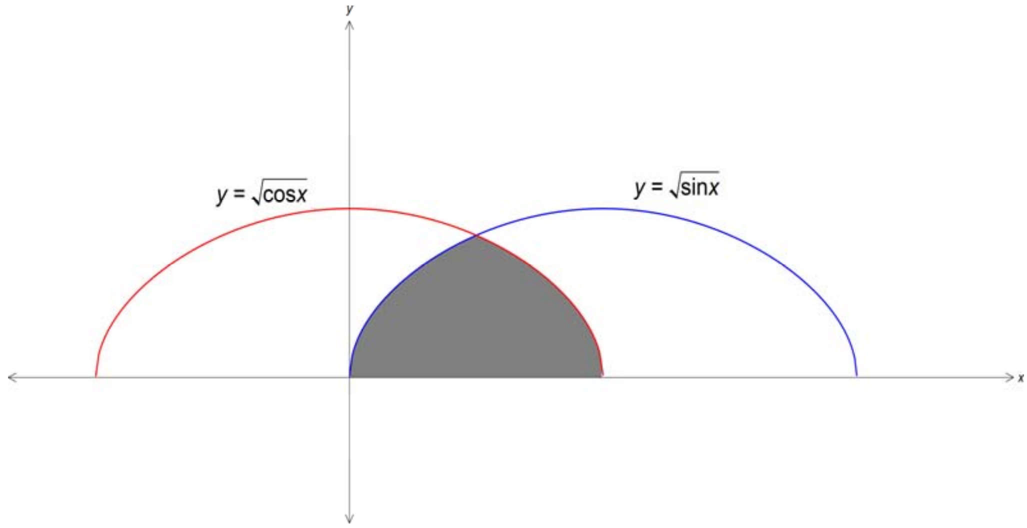
(a) A medicinal tablet is in the form of a disc with diameter 3.6 mm. The rate at which the tablet dissolves is given by $\frac{dr}{dt} = -k$, where r is the radius of the tablet at time t minutes and k is a positive constant.

(i) Show that $r = \frac{9}{5} - kt$. **2**

(ii) If the tablet dissolves completely in 12 minutes, find k. **2**

(iii) What does the value of k tell you about how the tablet dissolves? **1**

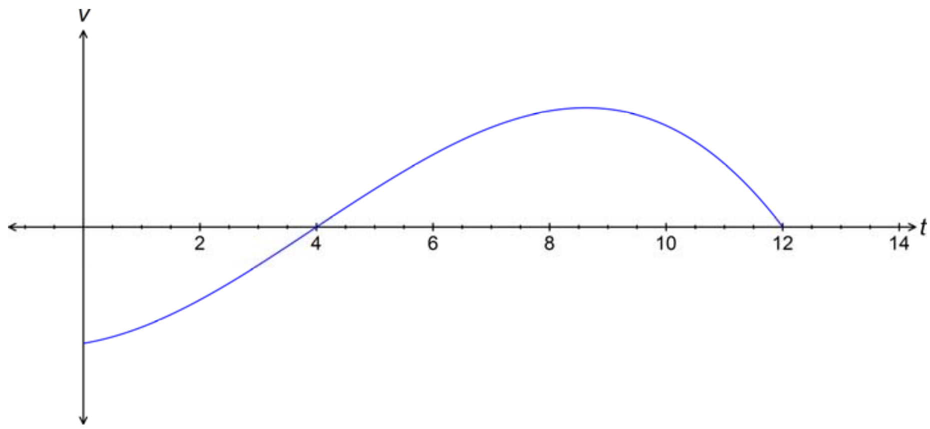
(b) Consider the shaded region bounded by the curves $y = \sqrt{\cos x}$ and $y = \sqrt{\sin x}$ and the x-axis.



(i) Show that the curves intersect at the point $\left(\frac{\pi}{4}, \frac{1}{\sqrt[4]{2}}\right)$. **2**

(ii) The region is rotated about the x-axis to form a solid. **3**
Find the exact volume of the solid.

(c) The graph shows the velocity-time graph of a particle.



(i) When is the particle stationary? **1**

(ii) When is it furthest from its starting point? **1**

(iii) Carefully sketch the graph of the acceleration of the particle. **3**

Question 16 (15 Marks) Use a NEW writing booklet.

(a) The population of two towns Lontown and Sydtown are given by $P_L = 1200e^{0.04t}$ and $P_S = 1500e^{0.025t}$ respectively, where t is the number of years elapsed since January 2000.

(i) In which year will the Lontown population exceed 1500? **2**

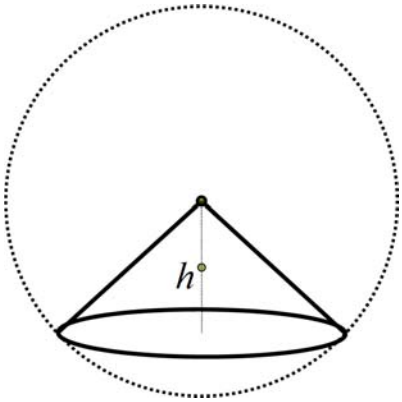
(ii) In which year will the rate of growth of the population in Lontown be double the rate of growth in Sydtown? **3**

(b) Saahil invests \$450 000 in an account which earns 8% per annum interest compounded annually. He intends to withdraw a fixed amount \$$M$ at the end of each year to live on during his retirement. He wishes to be able to do this for 20 years at which time the account will be empty. The amount of money in the account after n years is given by A_n.

(i) Show that $A_2 = 450000 \times 1.08^2 - M\left(1.08+1\right)$ **2**

(ii) Find the value of M that Saahil will have to spend each year. **3**

(c) The diagram shows a right cone inside a sphere. The vertex of the cone is at the centre of the sphere. The radius of the sphere is 9 cm and the perpendicular height of the cone is h cm.



(i) Show that the volume of the cone in terms of h is $V = \frac{\pi}{3}\left(81h - h^3\right)$ **1**

(ii) Find the maximum volume of the cone. **4**

End of Paper.

Mathematics Trial HSC 2012 Solutions

Q1. $1 - e^{-0.1} = 0.09516...$
$= 0.0952$ (3sf). (B)

Q2. $\dfrac{16 - a^2}{8 - 2a} = \dfrac{\cancel{(4-a)}(4+a)}{2\cancel{(4-a)}}$
$= \dfrac{4+a}{2}$ (C)

Q3. (A)

Q4. $a = +\frac{2}{5}$ $r = -\frac{1}{3}$. $S_\infty = \dfrac{2/5}{1 + 1/3}$
$= \dfrac{3}{10}$. (D)

Q5 $\dfrac{\sin x}{1} = \dfrac{\sin 106}{5}$
$x = \sin^{-1}\left(\dfrac{\sin 106}{5}\right)$ (A)

Q6. $\frac{d}{dx} x\sin x = x\cos x + \sin x$.
(D)

Q7. $P(\text{Blue}) = \dfrac{2}{7}$ (D)

Q8. $\ln a = \ln b + \ln c$
$\ln a = \ln bc$ (A)
$a = bc$

Q9. $(3 - \sqrt{5})^2 = 9 - 6\sqrt{5} + 5$.
$= 14 - 6\sqrt{5}$. (B)

Q10. $A\hat{B}D = 64°$ (base < 180s △ ABD)
$D\hat{A}B = 52°$ (sum < △ABD) (B)
$x = B\hat{C}D = 52$ (opposite < parallelogram are equal)

B - 1
C - 2
A - 3
D - 4
A - 5
D - 6
D - 7
A - 8
B - 9
B - 10

Question 11

(a) (i) $l = \frac{80}{360} \times 2\pi \times 3$

$= \frac{4\pi}{3}$ cm

1 — 0 marks for $120°$!!

(ii) $A = \frac{1}{2} r^2 \theta$

$3 = \frac{1}{2} \times 3^2 \times \theta$

$\theta = \frac{2}{3}$ radian

2 — MANY HAD IT IN DEGREES LOST 1 MARK

(b) $\frac{dy}{dx} = \frac{3}{3x-1}$

2

(c) $\int_{\pi/4}^{\pi/3} \sec^2 x \, dx = \left[\tan x \right]_{\pi/4}^{\pi/3}$

$= \tan \pi/3 - \tan \pi/4$

$= \sqrt{3} - 1$

2 — SOME LOST 1 FOR POOR APPROX.

(d) AP $a = 7$ $d = 3$

(i) $t_n = a + (n-1)d$

$t_{14} = 7 + 13 \times 3$

$= 46$ pages

1

(ii) $S_{14} = \frac{n}{2}(2a + (n-1)d)$

$= \frac{14}{2}(2 \times 7 + 13 \times 3)$

$= 371$ pages.

1

(iii) $p + 2p + 3p + \dots 14p = 630$

AP $a = p$, $d = p$

$S_{14} = \frac{14}{2}(2p + 13p) = 630$

$\frac{14 \times 15p}{2} = 630$

$p = 6$

3

(e) $\frac{dy}{dx} = 2e^{2x}$

gradient at $x = -2$ $\frac{dy}{dx} = 2e^{-4}$ — gradient of normal is $-\frac{e^4}{2}$.

At $x = -2$ $y = e^{-4}$

Equation of normal $y - e^{-4} = -\frac{e^4}{2}(x + 2)$

3

uestion 12

a) (i) $d^2 = 4^2 + 6^2$

$d = \sqrt{52}$ 1

$d = 2\sqrt{13}$ 1

(ii) $m_{AB} = -\frac{6}{4}$

$= -\frac{3}{2}$ 1

(iii) l has same gradient as AB

$\therefore$ equation is $y - 9 = -\frac{3}{2}(x+3)$ 1 using formula

$2y - 18 = -3x - 9$ 1 rearranging

$2y + 3x - 9 = 0$.

(iv) sub $(1,3)$ in l. 1

$LHS = 2\times3 + 3\times1 - 9$

$= 0 = RHS$.

(v) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ Equation of AB $y = -\frac{3}{2}x + 6$

$2y = -3x + 12$

$\underset{b}{2y} + \underset{a}{3x} \underset{c}{-12} = 0$.

$|3\times1 + 2\times3 - 12| = \frac{|\cancel{\ldots} - 12|}{\sqrt{4+9}}$

$= \frac{3}{\sqrt{13}}$

correct a,b,c 1

correct formula 1

(vi) Area = perp height × length of side AB

$= \frac{3}{\sqrt{13}} \times 2\sqrt{13}$

$= 6\ u^2$

(b) $\int_{3.1}^{3.5} f(x) \approx \frac{0.2}{3}(4 + 4\times5.6 + 3.2)$ 1 1

$= \frac{148}{75}$ ($\approx 1.97\dot{3}$).

(c) $P(3\text{ girls}) = \frac{18}{30} \times \frac{17}{29} \times \frac{16}{28}$ 1

$= \frac{204}{1015}$.

$P(\text{at least 1 boy}) = 1 - P(3\text{ girls})$

$= 1 - \frac{204}{1015}$

$= \frac{811}{1015}$ 1

Question 13.

(a) $px^2 - x + q = 0$

sum of roots $= -\frac{b}{a}$

$= \frac{1}{p} = -2+5$

$\therefore \frac{1}{p} = 3.$

$p = \frac{1}{3}.$ 1

product of roots $= \frac{c}{a}$

$= \frac{q}{p} = -2 \times 5.$

$\therefore \frac{q}{1/3} = -10.$

$q = \frac{-10}{3}$ 1

(b). $\sum_{n=0}^{4} (-2)^n = (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4$ 1 procedure

$= 1 - 2 + 4 - 8 + 16.$

$= 11$ 1

(c) Area $= \int_0^2 (x+2)^{1/2} - (x^2 - x)\, dx.$ 1

$= \int_0^2 (x+2)^{1/2} - x^2 + x \, dx$

$= \left[\frac{2(x+2)^{3/2}}{3} - \frac{x^3}{3} + \frac{x^2}{2} \right]_0^2.$ 1 integrate

$= \left(\frac{2 \times 8}{3} - \frac{8}{3} + 2 \right) - \left(\frac{2 \times 2^{3/2}}{3} \right).$

$= 2.78 \, u^2$ 1 final answer.

(d) (i) $y = x^2 e^x$ 1

$\frac{dy}{dx} = x^2 e^x + 2x e^x$ 1

$= x(x+2)e^x$

(ii) turning point when $\frac{dy}{dx} = 0$

ie $x(x+2)e^x = 0$ 1

$x = 0$ or $x = -2$.

$\frac{d^2y}{dx^2} = x^2e^x + 2xe^x + 2xe^x + 2e^x$

$= e^x(x^2 + 4x + 2)$

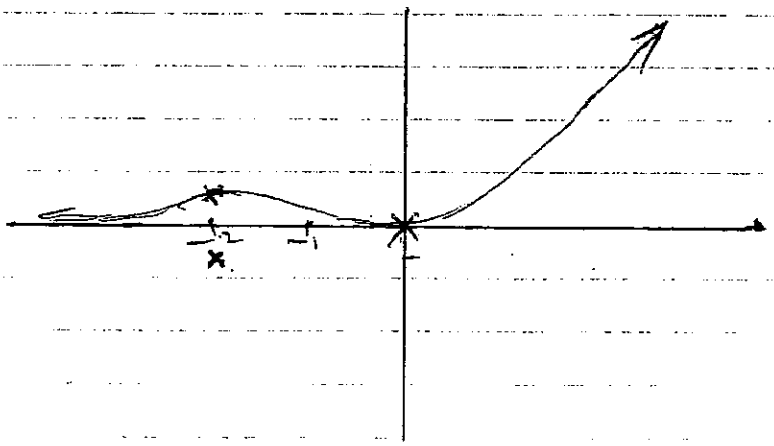
when $x = 0$ $y = 0$

$\frac{d^2y}{dx^2} = 2 > 0$ $\therefore$ local min at $(0,0)$. 1

when $x = -2$ $y = \frac{4}{e^2}$.

$\frac{d^2y}{dx^2} = e^{-2} \times -2 < 0$ $\therefore$ local max at $(-2, \frac{4}{e^2})$. 1

(iii) when $x = 0$ $y = 0$. only intercept. 1



shape 1
intercept 1

As $x \to \infty$ $y \to \infty$.
$x \to -\infty$ $y \to 0$.

$\int_0^1 x - x^2$

$\frac{x^2}{2} - \frac{x^3}{3}$

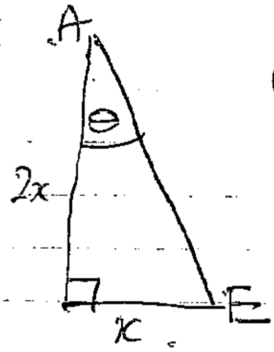
$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$

$\frac{x^3}{3} - \frac{x^2}{2} = \frac{1}{3} - \frac{1}{3}$

Question KF

(a)



(i) $AE^2 = (2x)^2 + x^2$

$= 5x^2$

$AE = \sqrt{5}x$

(ii) $\tan\theta = \frac{x}{2x}$

$\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\frac{1}{2}$

$= 27°$ (nearest degree)

$$\boxed{\begin{aligned} B\hat{A}E &= 90 - (60+27) \\ &= 3° \end{aligned}}$$

$\therefore$ Angle of depression is $63°$.

(b) (i) $A\hat{B}E = A\hat{C}D$ (given)

$B\hat{A}E = C\hat{A}D$ (common)

$\triangle ABE \,|||\, \triangle ACD$ (Equiangular)

(ii) $\cancel{\frac{x+3}{38/5} = \frac{6}{x}}$ $\qquad$ $\frac{x+8}{5} = \frac{4}{x}$

$\cancel{x(x+3) = \frac{228}{5}}$ $\qquad$ $x^2 + 8x - 20 = 0$

$x = 2 \quad x = -10$

$\cancel{5x^2 + 15x - 228 = 0}$ $\qquad$ $x = 2$

$\cancel{(5x \quad)(x \quad) = 0}$ or by ratios of corresponding sides

(c)(i) $y = \frac{1}{8}x^2$

$\frac{dy}{dx} = \frac{1}{4}x$

At $x = 8$ $\quad \frac{dy}{dx} = \frac{1}{4} \times 8$

$= 2$

Equation of tangent is $y - 8 = 2(x-8)$

$y - 8 = 2x - 16$

$2x - y - 8 = 0$

(ii) directrix is at ~~x=4~~ $y = -2$

at this point D $\quad 2x + 2 - 8 = 0$

$2x = 6$

$x = 3$

(iii) $M_{PF} = \frac{6}{8}$

$= \frac{3}{4}$

$M_{FD} = -\frac{4}{3}$.

since $M_{FD} \times M_{PF} = \frac{3}{4} \times \frac{-4}{3}$

$= -1$.

$P\hat{F}D = 90^\circ$

(d) $\frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+1}dx = \frac{1}{2}\ln(e^{2x}+1) + C$

Mathematics Trial HSC 2012 15 marks.

Question 15

$d = 3\cdot6\,mm$

$r = 1\cdot8\,mm$

$r = \frac{9}{5}\,mm$

(a) (i) $\frac{dr}{dt} = -k$

$r = -kt + c$ ✓ 1 mark (must have +c)

when $t=0$ $r = \frac{9}{5}$

$\frac{9}{5} = -k\times0 + c$ ✓ 1 mark (must show the substitution)

$\frac{9}{5} = c$

$\therefore r = -kt + \frac{9}{5}$

$\therefore r = \frac{9}{5} - kt$

(ii) $t = 12$ $r = 0$

$r = \frac{9}{5} - kt$

$0 = \frac{9}{5} - k\times12$ ✓ 1 mark (must show the substitution)

$12k = \frac{9}{5}$

$k = \frac{9}{60}$

$k = \frac{3}{20}$ ✓ 1 mark.

(iii) k tells you the rate at which the tablet dissolves is constant. ✓

(b) $y = \sqrt{\sin x}$

$y = \sqrt{\cos x}$

$\therefore \sqrt{\cos x} = \sqrt{\sin x}$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$ x is in Q1

$\therefore x = \frac{\pi}{4}$

when $x = \frac{\pi}{4}$ $y = \sqrt{\sin\frac{\pi}{4}}$ $y = \sqrt{\cos\frac{\pi}{4}}$

$y = \sqrt{\frac{1}{\sqrt{2}}}$ $y = \sqrt{\frac{1}{\sqrt{2}}}$

$y = \frac{1}{\sqrt[4]{2}}$ $y = \frac{1}{\sqrt[4]{2}}$

$\therefore$ the point of intersection is $\left(\frac{\pi}{4}, \frac{1}{\sqrt[4]{2}}\right)$

OR Use substitution

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sqrt{\sin\frac{\pi}{4}} = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt[4]{2}}$

$\sqrt{\cos\frac{\pi}{4}} = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt[4]{2}}$

(must show substitution)

Mathematics Trial HSC 2012 15 marks.

Question 15

$d = 3 \cdot 6$ mm

$r = 1 \cdot 8$ mm

$r = \frac{9}{5}$ mm

(a) (i) $\frac{dr}{dt} = -k$

$r = -kt + C$ ✓ 1 mark (must have +C)

when $t = 0$ $r = \frac{9}{5}$

$\frac{9}{5} = -k \times 0 + C$ ✓ 1 mark (must show the substitution

$\frac{9}{5} = C$

$\therefore r = -kt + \frac{9}{5}$

$\therefore r = \frac{9}{5} - kt$

(ii) $t = 12$ $r = 0$

$r = \frac{9}{5} - kt$

$0 = \frac{9}{5} - k \times 12$ ✓ 1 mark (must show the substitution)

$12k = \frac{9}{5}$

$k = \frac{9}{60}$

$k = \frac{3}{20}$ ✓ 1 mark.

(iii) k tells you the rate at which the tablet dissolves is constant. ✓

(b) $y = \sqrt{\sin x}$

$y = \sqrt{\cos x}$

$\therefore \sqrt{\cos x} = \sqrt{\sin x}$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$ ✓

$\tan x = 1$ x is in Q1

$\therefore x = \frac{\pi}{4}$

OR Use substitution

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sqrt{\sin\frac{\pi}{4}} = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt[4]{2}}$ ✓

$\sqrt{\cos\frac{\pi}{4}} = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt[4]{2}}$ ✓

(must show substitution)

when $x = \frac{\pi}{4}$ $y = \sqrt{\sin\frac{\pi}{4}}$ $y = \sqrt{\cos\frac{\pi}{4}}$

$y = \sqrt{\frac{1}{\sqrt{2}}}$ $y = \sqrt{\frac{1}{\sqrt{2}}}$

$y = \frac{1}{\sqrt[4]{2}}$ $y = \frac{1}{\sqrt[4]{2}}$

✓ $\therefore$ the point of intersection is $\left(\frac{\pi}{4}, \frac{1}{\sqrt[4]{2}}\right)$

(16) (a) (i) (2)

$$1200e^{0.04t} > 1500$$
$$e^{0.04t} > 1.25$$
$$0.04t > \ln 1.25$$
$$t > \frac{\ln 1.25}{0.04}$$
$$t > 5.57\ldots$$

$\therefore$ In 2005

Full marks given even if interpretation of the actual year was incorrect.

(ii) (3)

$$\frac{dP_L}{dt} = 2 \times \frac{dP_S}{dt}$$
$$48e^{0.04t} = 2 \times 37.5e^{0.025t}$$
$$\frac{e^{0.04t}}{e^{0.025t}} = \frac{75}{48}$$
$$e^{0.015t} = \frac{75}{48}$$
$$0.015t = \ln \frac{75}{48}$$
$$t = \frac{\ln \frac{75}{48}}{0.015}$$
$$= 29.75\ldots$$

$\therefore$ In 2029.

Same interpretation as (i) allowed

No marks if no attempt to differentiate.

(b) (i) (2)

$$A_1 = 450\,000 \times 1.08 - M$$
$$A_2 = (450\,000 \times 1.08 - M) \times 1.08 - M$$ ← This step was critical in earning full marks. Many took it for granted.
$$= 450\,000 \times 1.08^2 - 1.08M - M$$
$$\therefore A_2 = 450\,000 \times 1.08^2 - M(1 + 1.08)$$

(ii) (3)

$$\therefore A_{20} = 450\,000 \times 1.08^{20} - M(1 + 1.08 + 1.08^2 + \cdots + 1.08^{19})$$
$$= 450\,000 \times 1.08^{20} - M \cdot \frac{1.(1.08^{20} - 1)}{1.08 - 1}$$

But $A_{20} = 0$

$$\therefore \quad M = \frac{450\,000 \times 1.08^{20}}{\left[\frac{1.08^{20} - 1}{0.08}\right]}$$
$$= \$45\,833$$

OVER

(c) (i) (1)

$h^2 + r^2 = 81$

$\therefore r^2 = 81 - h^2$ ← Evidence of this was needed for the mark. Many manufactured the answer.

$V = \frac{1}{3}\pi r^2 h$

$\therefore V = \frac{1}{3}\pi\,(81 - h^2)h$

$V = \frac{1}{3}\pi\,(81h - h^3)$

(ii) (4)

$\frac{dV}{dh} = \frac{\pi}{3}(81 - 3h^2)$

$\frac{dV}{dh} = 0$ when

$\frac{\pi}{3}(81 - 3h^2) = 0$

$3h^2 = 81$

$h^2 = 27$

$h = 3\sqrt{3}$ (since $h > 0$)

$\frac{d^2V}{dh^2} = \frac{\pi}{3} \cdot -6h$ ← One mark deducted if max. not established.

when $h = 3\sqrt{3}$ $\frac{d^2V}{dh^2} < 0$

$\therefore$ a maximum

$\therefore$ max $V = \frac{\pi}{3}\left(81 \times 3\sqrt{3} - (3\sqrt{3})^3\right)$ ← Any exact form of answer accepted.